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A BRIDGE MANAGEMENT STRATEGY BASED ON FUTURE RELIABILITY AND SEMI-MARKOV DETERIORATION MODELS

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ABSTRACT

This paper introduces the outline of a bridge management strategy based on the prediction of future bridge reliability using a semi-Markov deterioration model. This approach applies equally well to single bridges and to a whole network, and is expected to be implemented in the Bridge Management System (BMS) of the Autonomous Province of Trento (APT). More in detail, the model assumes the lifespan of a bridge to be divided into five conventional condition states, and waiting times in each state are random variables with known distributions. Mean waiting times and probability distribution parameters are currently estimated based on the information stored in the APT's BMS. Monte Carlo simulations are used to calculate the cumulative waiting time distributions, from which the semi-Markov transition probability matrices are derived. The transition probabilities are age dependent: older bridges have a higher probability of deteriorating to the next condition state in the given time interval. Once calibrated, the deterioration model allows calculation of the time variant capacity function, in terms of probabilistic initial capacity and degradation function. The prioritization is based on the principle whereby priority is given to those actions that, within a certain budget, will minimize the risk of occurrence of an unacceptable event in the whole network during the considered time interval. Sample results of the prioritization as applied to the APT stock are then discussed.

1. INTRODUCTION

Bridge Management Systems (BMSs) are tools designed to help bridge managers keep track of the bridge stock characteristics, conditions and serviceability. In addition to this, the most recent BMSs feature structural reliability assessment and decision making tools capable of analyzing maintenance plans and total life-cycle costs. The fundamental goal of these instruments is to allow the owner to establish an effective operation strategy for the stock as a compromise between many conflicting technical and social objectives, such as: minimization of life-cycle cost, minimization of probability of failure, maximization of network performance.

The Autonomous Province of Trento (APT), Italy, has recently adopted a Bridge Management System entirely based on these concepts. The system operates on the web, and includes modules for condition state evaluation, safety assessment and prioritization. Condition appraisal is based on visual inspection, and acknowledges the general rules of the AASHTO Commonly Recognized Standard Element system [1].

A conservative reliability assessment procedure is carried out for each bridge in the stock and is based on the sole inspection data. When the condition of the bridge calls for a more detailed evaluation, its reliability is evaluated using multi-step procedures that follow the BRIME research project guidelines [2]. These procedures are of increasing refinement and include explicit probabilistic analyses at the higher evaluation steps. The prioritization is based on the principle whereby priority is given to those actions that, within a certain budget, will minimize the risk of occurrence of an unacceptable event in the whole network during the considered time interval, usually assumed equal to 50 years. The optimization module for the choice of an effective maintenance strategy has been recently developed and is now undergoing a test program, but is not yet included in the APT BMS.

In this paper, we discuss the operation of this system, we present the models used to predict future bridge performance and we illustrate some sample results obtained from the prioritization process.

2. THE APT BRIDGE STOCK AND MANAGEMENT SYSTEM

The APT is a mountainous region in the Italian Alps. Currently, APT owns and manages approximately 2340 kilometres of roadways and 936 bridges. As to range of bridge types and ages, the stock may be considered quite similar to other European stocks: most of the bridges were constructed or reconstructed after the Second

World War, the age distribution diagram showing a peak in the 70's. Also, reinforced concrete, regular and pre-stressed, is by far the most widely utilized construction material, covering more than 74% of the entire stock. As to types, 65,1% of APT bridges are RC or PRC simply-supported or continuous beams, 10% are RC arches, 19,9% are simple concrete or masonry arches, while only the remaining 6% includes steel and steel-concrete composite bridges.

The main characteristics of the Bridge Management System are:

- the system gives the owner not only a clear indication of the condition of each bridge, but also of its safety level, expressed in terms of a reliability index;
- for each bridge a priority index is calculated for scheduling maintenance activity;
- all information is provided in real-time;
- the system is fully operative on the web; inspectors and evaluators upload the results of the conditions and safety assessments through a web-based interface; the manager can access the results of the analysis through the same web interface;
- all the subjects involved in management operations can directly interact with the system: DoT managers, DoT inspectors, professional engineers involved in the assessment procedure, external consultants;
- maintenance and upgrade of the system are continuous and transparent to the users.

The BMS is based on an SQL database that collects the data for the whole stock of bridges. The data is organized in project level data and network level data. Project level data are:

- Inventory Data;
- Condition State Data;
- Reliability Data.

Inventory Data includes all the information related to bridge identification, geographical location and features, administrative issues, construction and previous retrofits.

Condition State data give a measure of the type and severity of the deterioration of the structure. The aim of the condition assessment of bridge structures is to detect whether a deterioration process is going on, and, if so, to evaluate the degree of deterioration, with respect to the bridge in its original conditions. According to [1], each element of the bridge is associated with a condition state. The number of reference states foreseen by the procedures varies from 3 to 5, depending on the element. Each state is labelled with a number, whereby 1 always indicates the undamaged situation while the maximum value refers to the most severe damage situation. Since the condition state is univocally stated in the BMS procedures, to which all inspectors must conform, this can be seen as an objective and quantitative measure of damage. An overall condition index of the bridge is also calculated by weighting all the elements condition states according to their importance, as better explained in the next Section.

Reliability Data refers directly to the load carrying capacity of the bridge and consists of a set of reliability indices, each associated with an ultimate limit state and a specific Structural Unit or substructure.

Network level data includes all the information that is not related to a specific bridge, but is relevant to the whole stock or to a group of bridges having the same structural system and/or material. Network level data are for example the global scheduling of maintenance actions, the price list of interventions that defines the cost model and the deterioration rates of the Markovian matrices that define the deterioration model of each Standard Element.

Inventory Data, Condition State Data and Reliability Data are used as input for the applications that perform network level analysis operations and the results themselves are recorded into the database.

3. DETERIORATION MODEL

There have been many attempts to develop analytical models representing the degradation of bridge structures starting from the knowledge of the deterioration process of materials [3]. These models are typically very complex, as they attempt to represent physical and chemical processes using specific models for each different degradation cause. Moreover, they have to account that the actual condition of a bridge is the consequence of many concurring degradation processes. For these reasons, to obtain an effective and reliable mathematical degradation model is a very difficult task.

A different approach defines the degradation model by analyzing the condition state variation with time, on the basis of historical and experimental data. In this way, the manifold concurrent degradation causes are implicitly taken into account and there is no need to define specific mathematical models. This allows calculation of an appropriate degradation curve for each bridge, reflecting all the specific factors that have caused its degradation.

In the APT-BMS, there is currently no historical data available. To define the condition variation over time, all the BMS bridges have been grouped as to their structural type and material used. An overall condition index for each bridge in has been calculated according to the following four-step procedure:

- Step 1: the numerical value associated with the condition state of each element is normalized to a condition index (CI) spanning from 1 to 5;
- Step 2: the CI of each group of elements is calculated as the maximum normalized value obtained in Step 1;
- Step 3: a specific normalized weighting is assigned to each group of bridge elements (such as: superstructure, substructure, equipment...), according to their structural importance;
- Step 4: the overall bridge CI is calculated as a sum of the weighed CI.

The resulting CI is a real number in the interval [1,4], 1 representing the design state and 5 the most severely damaged situation. It should be noted that this procedure associates with each element a condition index proportional to the value of the numerical label of its condition state. Therefore, this method implicitly assumes that the damage level is proportional to the discrete value of CS. It is understood that a more refined approach should consider weighting these values according to the actual damage severity.

By plotting the overall condition state as a function of the bridge age, two degradation curves can be defined for optimal environmental conditions and bad environmental conditions. For example, Figure 1 shows the results for simply supported concrete bridges. The degradation curve of each bridge is obtained through a set of explanatory variables that represent its specific environmental conditions and will be between the two extremes.

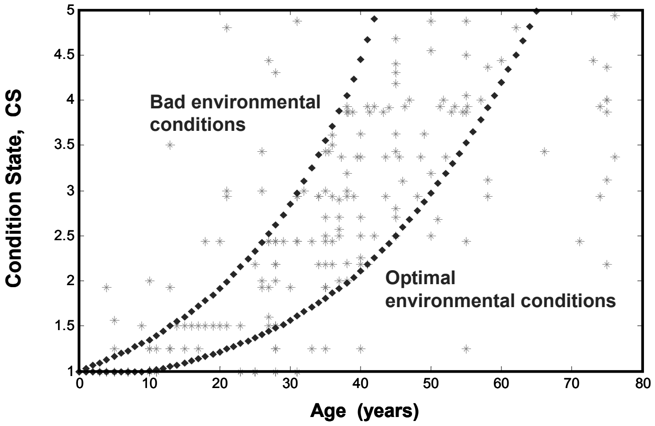

Figure 1. Simply supported concrete bridges's CS over age.

With this approach, bridge inspection data is used to develop probabilistic models that predict the future bridge condition. It should be noted that even though the bridge condition ratings are assigned following a standard procedure, the subjective judgments of bridge inspectors is not completely ruled out and thus the trend may reflect inherent human bias. Accurate predictions are essential for effective MR&R decision making, therefore probabilistic deterioration methods are used to characterize bridge deterioration.

4. MODELLING BRIDGE DETERIORATION AS A SEMI-MARKOV PROCESS

Markov chains models have been used extensively to predict future bridge conditions. Several methods for estimating transition probabilities from the available bridge inspection data have been suggested in the literature [5,6].

A common assumption is that transition probabilities do not depend on the facility's age, i.e., that transition probabilities are homogeneous. In the APT-BMS the degradation process has been developed and is currently modelled with Static Markov Chains. Non-stationary Markov chains, in which transition probabilities depend on the time spent in an initial condition state, appear to be more realistic [7,8,9].

A semi-Markov process is defined as a process that changes states in accordance with a Markov chain but remains in a state for a definite period of time. The time needed for a facility to pass from an initial condition state to its next specified condition state is defined as transition time. This amount of time, also called waiting time or time-in-state, is assumed to be a random variable.

Let $T_1, T_2, ..., T_n$ be random variables denoting the waiting times for each condition state {1, 2,…,n} respectively, where n is the number of condition states. T_i is a random variable with continuous and positive density function $f_{Ti}(t)$, cumulative density function $F_{Ti}(t)$ and survival function $S_{Ti}(t)$. Let T_{1k} be a random variable denoting the cumulative time of permanence in states {1, 2,…,k}, i.e., the time spent by the process to go from state *1* to state *k*.

$$T_{ik} = \sum_{j=1}^{k} T_j \qquad k = 1,2,...,n-1$$

The corresponding probability distributions are denoted as $f_{T1k}(t)$, $F_{T1k}(t)$, $S_{T1k}(t)$. As introduced by Kleiner [10], it can be shown that if the process is in state *i* at time *t*, the conditional probability that it will pass to the next state in the time step Δt can be expressed as follows:

$$p_{\mathrm{i,i+1}}(\mathrm{t}) = \frac{\mathrm{f}_{\mathrm{T1i}}(t)}{S_{T1i}(t) - S_{1(i-1)}(t)} \qquad i = 2,3...,n-1$$

This equation provides all the transition probabilities needed to obtain the transition probability matrix *P(t)* for the semi-Markov process. These transition probabilities are time-dependent and the process is non-stationary. This means that the older the asset is, the higher the likelihood is of deterioration to the next state in a given period of time.

Given the transition probability matrix, the condition state vector collecting the probabilities of being in a given state, can be obtained as:

$$A(\mathrm{t}+\mathrm{k}) = A(t)\prod_{i=0}^{k-1} \mathrm{P}^{\mathrm{t+i,t+i+1}}$$

where $P^{t,t+1}$ denotes the transition probability matrix from time *t* to time *t+1*. Figure 2 shows an example of the condition state progression over time for a simply supported concrete bridge, i.e., the prediction over time of the probability that the bridge will be in a given condition state, assuming that at time *t=0* its condition state is equal to 1. In this example Weibull probability distributions are assumed to represent the duration in time in each condition state.

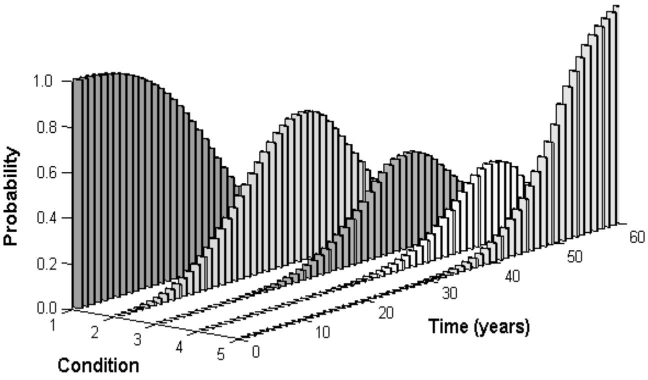

Figure 2. Progression of the condition state vector over time for a simply supported concrete bridge.

Once the deterioration process is modelled, a comparison can be made between the mean estimated values of condition state over time and the observed values (Figure 3). Under the hypothesis that no maintenance is performed, the estimated mean condition state values show good correspondence with the observed degradation curve when the bridge is in optimal environmental conditions. The time variant condition state vector *A(t)* for a simply supported concrete bridge under non-optimal environmental conditions is calculated as an empirically defined function of the condition state vector under optimal environmental conditions, and the explanatory variables that mostly affect the deterioration rate of a bridge, i.e. daily heavy vehicles occurrence, number of days with freezing temperature in a year, and so on. The actual curve used for each bridge will be between the optimal- and the worst-conditions curve, and will be defined on the basis of specific values of the explanatory variables.

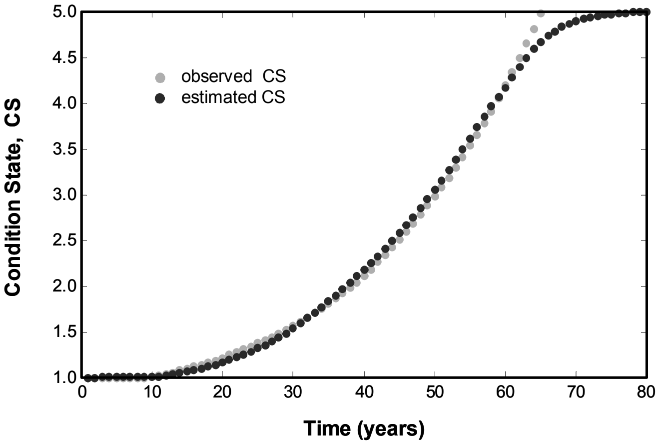

Figure 3. Progression of the bridge condition state vector over time.

The condition state vector *A(t)* is used for estimating the total life-cycle costs and the cumulative-time probability of failure for each bridge in the stock, as explained in Section 7.

5. ACTION MODEL

The mathematical model developed to represent actions and their effects defines four actions: Routine Maintenance, Repair, Rehabilitation and Reconstruction. Routine Maintenance actions comprehend protection measures, ordinary maintenance operations and minor repairs. Repair actions comprehend structural repair operations, equipment repair and substitution. Rehabilitation actions comprehend major repair operations, performed on the whole structure both on structural and on equipment elements.

Actions are meant to extend the life-cycle of a bridge by positively affecting its degradation process. In the current implementation of the system the effects of maintenance actions are modelled by the Markov approach: for each actions a modified transition matrix is defined. When a maintenance action is performed, the corresponding transition matrix is applied to the known transition states to model the effects of the action. Guigner and Madanat [9] proposed a method to estimate the transition probability matrix when a rehabilitation action is performed. In this method though, the transition matrix is obtained without considering the effects of the actual actions performed and there is no differentiation between repair and rehabilitation actions.

In the method proposed in this paper, which is explained in detail in [11], the effects of the maintenance actions are taken into account by introducing a variation in the degradation curve, obtained by calculating the mean value of the condition state vector *A(t)* (Figure 4). In the method proposed, Routine Maintenance and Repair actions stop the degradation process for a given number of years, Rehabilitation takes the structure to a lower CS (i.e. it takes the bridge condition back in time for *n* years) and stops the degradation process for a given number of years, while Reconstruction takes the structure back to CS = 1. The definition of the effects of these actions has been implemented through the use of parameters, to provide a more flexible model.

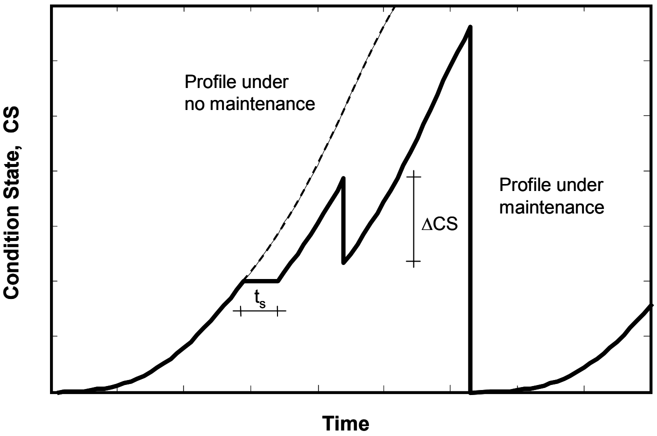

Figure 4. Maintenance actions model.

6. MAINTENANCE STRATEGY AND PRIORITIZATION ANALISYS

In the field of bridge management, several studies have investigated the maintenance strategy optimization (e.g. Ref. [12,13,14]), but have a number of limitations (e.g. they work with deterministic damage models or with failure criteria that are not reliability based). In recent years, probabilistic optimization methods have been increasingly used in maintenance models (e.g. Ref [15,16]). In several recent studies, (e.g. Ref. [17,18]) maintenance models have been developed for steel and concrete bridges based on reliability and minimum life-cycle cost.

In this paper a simple cost benefit analysis is presented for the project-level choice of a particular maintenance strategy and for the network-level prioritization of the selected strategies. The results obtained for three different structures are illustrated and discussed.

A set of possible maintenance actions is defined (e.g. routine maintenance, repair, rehabilitation, replacement) and then a given number of maintenance programs is considered. For each maintenance program and for each bridge a priority index is calculated as follows:

$$\alpha = \frac{\Delta P_X(t_L)}{\Delta C} = \frac{P_X(t_L) - P_{X|a}(t_L)}{\Delta C}$$

where $P_X(t_L)$ is the bridge cumulative-time probability of failure over the duration $(0, t_L]$ multiplied by an importance factor that in the current implementation depends on failure mode, bridge dimensions and average daily traffic, *a* is the considered maintenance program, ΔC is the actualized life-cycle cost associated with the action implementation.

For each maintenance program, the prioritization module calculates the cumulative-time probability of failure, the importance factor and the life-cycle costs. Mori and Ellingwood [19] first proposed a time-variant method for directly evaluating the cumulative-time failure probability of the series system. In Mori and Ellingwood's formulation, the degradation function is not statistically defined. A consequence of this assumption is that the standard deviation of the bridge capacity decreases with time, and this is apparently against common experience. To overcome this limit, a probabilistic degradation model is proposed. Assuming that the system reliability is dominated by a single limit state, and that this limit state can be formulated as the difference between a capacity R and a demand S, the cumulative-time probability of collapse can be formulated as:

$$P_F(t_L) = 1 - \int_0^{+\infty} \exp\left(-\lambda \left[t_L - \int_0^{t_L} \left\{ \int_0^1 F_S(r(1-\delta)) f_\delta(\delta) \mathbf{d}\delta \right\} dt \right] \right) f_{R_0}(r) dr$$

where λ and F_S are the mean occurrence rate and the cumulative distribution function of the demand S, δ is a probabilistic capacity degradation function depending on the time variant vector $A(t)$, which collects the condition states of the bridge, and f_{R0} is the probability density function of the baseline capacity R_0 assumed in design.

The life-cycle cost associated with a maintenance program is estimated as the sum of different costs: inspection cost, maintenance actions cost and failure cost. The inspection cost changes with type, size, accessibility of the structure and it is assumed to be time-independent. The maintenance actions cost (routine maintenance and rehabilitation) is estimated on the basis of routine maintenance or rehabilitation unit costs of bridge elements and depend on their condition states. Reconstruction cost is estimated as euros per square meter of deck area, and it depends on the typology of the elements composing the bridge. The failure cost takes into account all the structural and functional costs associated with a potential failure, included indirect costs and social costs. All future costs are actualized using the official financial discount rate published yearly in the APT Construction Pricelist Bulletin.

In the APT-BMS, the maintenance programs considered for the priority index calculation are:

- 0 (zero): do-nothing (routine maintenance only);
- A: rehabilitation intervention at time 0;
- B: reconstruction intervention at time 0.

For each maintenance program, the bridge deterioration process is evaluated and the corresponding time-cumulative probabilities, life-cycle costs, rehabilitation and reconstruction priority indexes are calculated. At project level the best intervention program is the one with the highest priority index, while network level prioritization is made by ordering all the best project level programs: the first intervention performed will be the one with the highest index α.

In the following we present the results obtained for three sample bridges in the APT stock. Table 1 reports a summary of the outcome of the prioritization analysis, automatically carried out by the BMS for the three reference maintenance programs.

The SP65 bridge on the Maso river has a simply supported concrete structure and is a common type of bridge in the APT stock. The structure shows minor deterioration of the beams, including localized concrete cover spalls, mostly due to an inefficient drainage system. In this case, strategy A (rehabilitation at time 0) has the highest priority index, as shown in Table 1.

The Canova Viaduct carries a 4-lane highway which represents one of the most critical road connections in the region, with an average daily traffic of over 15000 vehicles. The main structure is 686m long and 17.70m wide, and has 34 simply supported spans of variable length. The bridge dates from 1978 and shows signs of advanced deterioration at the cross-beams, resulting in some cases in the failure of the post-tensioning system. These faults are due to poor design in detail and execution. As shown in Table 1, the risk associated with the do-nothing maintenance program is relatively high. Rehabilitation is ranked as the most cost-effective action. However, reconstruction is also associated with a high priority, higher than that calculated for the rehabilitation of the SP65 bridge.

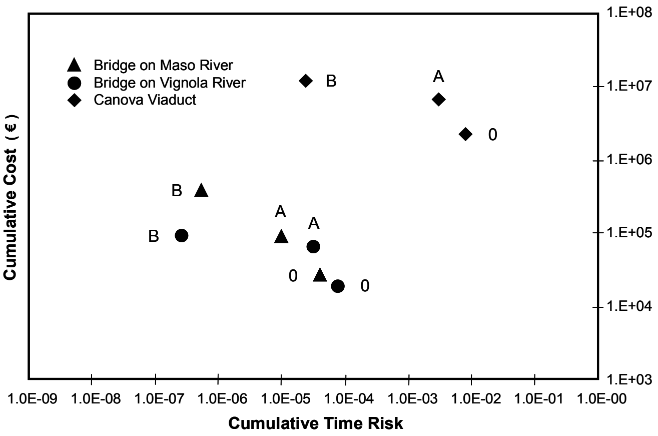

Figure 5. Cumulative time risk and total cost for three bridges in the APT stock.

SP12 bridge on Vignola river is a minor bridge serving a local road with an average traffic of a few vehicles per day and consisting of a single simply supported 5m-wide and 10m-long span. Given the minor importance of the bridge, the cumulative-time risk evaluated by the system is relatively low, as reported in Table 1. However, the cost of any potential action is also very low, and for this reason the resulting priority indexes are comparable with those of the Canova Viaduct.

SP65 Bridge on Maso River			
	0	A	B
P_X	$4.17\cdot10^{-5}$	$1.02\cdot10^{-5}$	$5.96\cdot10^{-7}$
Cost [€]	26300	83700	369280
ΔP_X	-	$3.15\cdot10^{-5}$	$4.11\cdot10^{-5}$
ΔC [€]	-	57000	343000
α [€$^{-1}$]	-	$5.49\cdot10^{-10}$	$1.19\cdot10^{-10}$

Canova Viaduct			
	0	A	B
P_X	$8.00\cdot10^{-3}$	$3.08\cdot10^{-3}$	$2.51\cdot10^{-5}$
Cost [€]	2069000	6733000	11893000
ΔP_X	-	$4.92\cdot10^{-3}$	$7.97\cdot10^{-3}$
ΔC [€]	-	4660000	9820000
α [€$^{-1}$]	-	$1.05\cdot10^{-9}$	$8.11\cdot10^{-10}$

SP12 Bridge on Vignola River			
	0	A	B
P_X	$7.80\cdot10^{-5}$	$3.31\cdot10^{-5}$	$2.79\cdot10^{-7}$
Cost [€]	20450	72300	98200
ΔP_X	-	$4.49\cdot10^{-5}$	$7.77\cdot10^{-5}$
ΔC [€]	-	51900	77800
α [€$^{-1}$]	-	$8.66\cdot10^{-10}$	$9.99\cdot10^{-10}$

Table 1. Outcome of the prioritization analysis performed on three bridges in the APT stock.

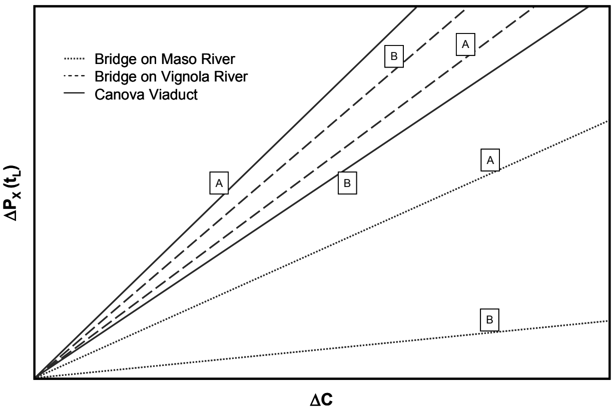

Figure 6. Graphical representation of the priority index.

In this case, replacement is ranked as the most cost-effective action. Figure 5 shows the cumulative time risks and the total costs for the three bridges considered. A bi-logarithmic scale is used due to the great difference between the Canova Viaduct and the other two bridges.

Figure 6 is a graphical representation of the priority indexes: each line corresponds to a maintenance program and its gradient is its priority index. The maintenance programs are thus ordered from left to right, from the one with the highest priority index to the one with the lowest.

7. CONCLUSIONS

In this paper an approach is presented to model the bridges deterioration based on observational data. The use of semi-Markov chains to characterize the deterioration process in a probabilistic way has led to a realistic prediction of the bridges condition state variation in time, even though a longer data history would be of great help for improving the results accuracy. The environmental conditions that mostly affect the degradation rate of a bridge are represented by a set of explanatory variables that modify the deterioration process. In this way each bridge has a specific probabilistic degradation curve. The effects of the maintenance actions are taken into account by introducing a stop or a negative jump in the degradation curve. For a given maintenance program, the deterioration of a bridge in terms of condition state variation in time is used to calculate the bridge cumulative-time probability of failure and the life-cycle costs. A cost-benefit analysis is performed in order to select the best project-level maintenance strategy (among those considered), and the highest intervention priority at network level. The prioritization analysis also considers the importance of a bridge in terms of geometrical dimensions and average daily traffic.

The results of the cost-benefit analysis automatically carried out by the BMS have been presented for three sample bridges of the stock. A drawback of this approach is that only a limited number of maintenance options is considered. In addition, the priority indices evaluated depend on the reference intervention scenario chosen. As a consequence, for a given bridge the selected maintenance strategy may not be optimal.

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